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FACULTY WORKING
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An Integration of Random Coefficient and
Errors-in-Variables Models for Beta Estimates

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An Integration of Random Coefficient and Errors-in-Variables Models for Beta Estimates

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Abstract

Based upon both theoretical and empirical arguments, the market model has been specified as a random coefficient and errors-in-variables (RCEV) rates of return generating process. The impacts of measurement errors associated with market rates of return and risk-free rate on the estimated beta coefficient and estimated random coefficient parameters are analyzed in detail. It is shown that the beta coefficient obtained from the RCEV can be decomposed into: (1) true component, (2) bias due to measurement errors, (3) bias due to specification bias, and (4) interaction bias.

I. Introduction

The main purpose of this paper is to show how a mixed random coefficient and errors-in-variable model [RCEV] can be used to interpret empirical results associated with beta estimates in a more realistic manner. It is shown that the most generalized beta estimate can be decomposed into four components. It is also shown that the measurement errors in the explanatory variable of risk-premium type of market model can bias the estimated beta and the estimated random coefficient parameters associated with beta estimate. The criteria for using either fixed coefficient or random coefficient type of market model in estimating the systematic risk are derived by reference to both bias and mean square error. Finally, the implications of these components to cost of capital estimates, investment analysis and portfolio management are discussed in some detail.

In the second section, the theoretical and empirical literature of capital and pricing model [CAPM] is reviewed. In the third section, possible measurement error components of both the market rates of return and risk-free rates of interest are defined. In the fourth section, the reason for using a random coefficient instead of a fixed coefficient market model is justified in accordance with the specification analysis developed by Theil (1971) and the multi-factor rate of return generating process discussed in the second section. The measurement errors of both market rates of return and risk-free rates are then introduced to the random coefficient market model. The impact of these measurement errors on the estimated random beta coefficient is then analyzed in detail. In the fifth section, the effects of measurement errors associated with excess market

rates of return on the estimated random coefficient parameters are analyzed in accordance with the technique developed by Griliches and Ringstad (1970). Possible implications of the results developed in this section on some well-known empirical research such as Fama and Macbeth's (1973) and Treynor and Mazuy's (1966) empirical results are also discussed. Finally, results of this study are summarized and concluding remarks are indicated.

II. Capital Asset Pricing Model: A Brief Review

There are several approaches to the determination of capital asset prices under conditions of uncertainty. The mean-variance model was developed by Markowitz (1952), Sharpe (1964), Lintner (1965) and Mossin (1966), while the state preference model originated from Arrow (1964) and Debreu (1959). Most recently, Ross (1976, 1977) has used an arbitrage theory to derive a multi-index model for capital asset pricing. Ross has argued that the arbitrage approach can be regarded as a compromise between the mean-variance approach and the state preference approach in determining the price of capital assets.

Specifying an individual expected utility function which contains the mean and the standard deviation of the returns of all assets in the investment opportunity set, Sharpe (1964) derived the model for capital asset pricing (CAPM). In deriving the CAPM, Sharpe has also assumed the existence of the risk-free rate of interest. The CAPM is defined as

$$R_j = R_F + \beta_j (R_m - R_F) , \quad (1)$$

where R_j = expected rate of return on the j^{th} asset,

R_F = risk free rate of interest,

R_m = rate of return on a "market portfolio" consisting of an investment in every asset outstanding in proportion to its value,

$\beta_j = \text{cov}(R_j, R_m) / \sigma^2(R_m)$ = the systematic risk of the j^{th} asset.

$\sigma^2(R_m)$ = variance of market rates of return.

Mossin (1966), Jensen (1972), and others have derived the economic implications of the single period CAPM from the general equilibrium framework; they are

- (i) For $n-1$ risky assets we have $n-1$ demand relationships; in addition, we also have a wealth constraint. These n equations formulate a complete demand system.
- (ii) In equilibrium with homogeneous expectations by all investors, prices must be such that each individual will hold the same percentage of the total outstanding stock of all risky assets.
- (iii) The CAPM is independent of the initial holdings in the individual assets, assuming perfect markets.
- (iv) The CAPM is free from aggregation problems.

Although the CAPM is an elegant method used in investigating portfolio performance, valuation theory, determination of the "cost of capital" and corporate investment decisions etc., it still faces many theoretical and empirical problems as indicated by Roll (1977), Ross (1976, 1977) and others.

Theoretically, the CAPM is based upon a mean-standard deviation utility analysis. In accordance with Tsiang (1972) and Levy and Markowitz (1979), this analysis is justified if and only if either

- (i) the investor's utility function is quadratic or (ii) the risk

aversion investors regard the uncertain outcomes as all normally distributed and nonsatiation in utility or (iii) the aggregate risk taken by the individual is small compared with his total wealth, including his physical, financial and human wealth.

By assuming that R_{jt} , R_{mt} and R_{Ft} are stationary, Jensen (1968, 1969) has shown the ex-post relationship of (1) as indicated in equation (2) can be used to estimate beta coefficient and Jensen performance measure.

$$R_{jt} - R_{Ft} = \alpha_j + \beta_j (R_{mt} - R_{Ft}) + \varepsilon_{jt} \quad (2)$$

where ε_{jt} is a disturbance term. The subscript t indicates the time series observations of each variable. Theoretically, α_j should be zero. However, Jensen showed that the estimated can be used to measure the investment performance of a portfolio (or security). A set of multiperiod data are generally used to estimate the single period model as indicated in equation (2). Roll (1969) has argued that the main problem in applying the one period model to time series data is the neglect of the change in wealth over time. This situation will make the systematic risk estimate either biased or nonstationary. Empirically, the market model in risk premium form [MMRPF] also faces errors-in-variables, specification, the time horizon and the random coefficient problems.²

According to the measurement and specification analyses, there exist three sources of bias: (1) the single period bias; (2) MV utility function; and (3) one factor model bias. The single period bias is due to the employment of multi-period data in a single period model. Jensen (1968, 1969) addresses this with an ex post factor which he then sets to zero expectation. The utility function bias is due to the fact that the utility function is generally assumed to include only two argu-

ments, the mean and the standard deviation of the rate of return. The one factor model bias asserts that the one factor specification is too simple to explain the real world. The one factor model has been extended to two-factor, three-factor and four-factor models as indicated in appendix A.

The review of this section has indicated that the rates of return generating process may well be a multi-index model instead of a single index model. Hence beta coefficients estimated by a simple regression market model may well be subject to the problem of omitted variables. Further, since the omitted variables are stochastic in the Fama-Macbeth and Merton models, the random coefficient phenomenon may be applicable, as found by Fabozzi and Francis (1978), Sunder (1980) and Lee and Chen (1980). The relationship between the random coefficient and omitted variables will be discussed in section four.

Although the measurement error problem of the CAPM has been investigated by Roll (1969), Friend and Blume (1970), Miller and Scholes (1972) and Lee and Jen (1978) etc., systematic research based upon a mixed model of both the errors-in-variables model and random coefficient model still remains to be done.

In order to begin such an investigation, the empirical model as indicated in equation (2), to be employed in this paper is now justified.

There are three different approaches used to estimate the systematic of CAPM. (a) Regressing R_{jt} on R_{Ft} and R_{mt} without any restriction, (b) Regressing $(R_{jt} - R_{Ft})$ on $(R_{mt} - R_{Ft})$, as indicated in equation (2), and (c) Regressing R_{jt} on R_{mt} by assuming R_{Ft} is constant over time.

From econometric theory, it can be easily shown that approach (a) will lose efficiency and obtain an estimator different from the con-

strained estimator as defined by the theoretical relationship of equation (2). Approach (c) does not consider the random nature of R_{Ft} , and it potentially induces specification error, as shown by Roll (1969). In sum, the systematic risk which was obtained from both (a) and (c) are generally different from that of (b). Therefore, we will use approach (b) as the basic specification in this paper.

III. Possible Error Components of Market Rate of Return and Risk Free Rate of Interest.

One error source is the measurement error resulting from tax and transaction cost effects. Brennan (1970) has derived a CAPM with corporate and personal taxes; it is shown that these taxes are important components in estimating the systematic risk. Milne and Smith (1980) have theoretically shown that the transaction cost is one of the important components in determining the systematic risk. These two studies imply that both tax and transaction cost should be included in the theoretical and empirical determination of capital asset pricing.

Another source of measurement error is the use of proxy variables. In general, the New York Stock Exchange (NYSE) average is used as a proxy of R_m and the monthly 90 day treasury bill rate is used as a proxy of R_F . A main source of measurement error in R_m is that the NYSE average only includes a subset of the market portfolio. The possible sources of measurement error in R_F are that the treasury bill rate is risk-free only in the default sense, and in the real world, investors cannot borrow and lend an unlimited amount at an exogenously given risk-free rate of interest R_F . Additional discussion of the measurement errors of R_m and R_F can be found in Lee and Jen (1978), Brennan (1971) and Roll (1969) etc.

For simplicity, we convert the CAPM notation to the more standard econometric notation, and thus we will rewrite (2) as³

$$v_{jt} = \beta_j u_{mt} + \varepsilon_{jt} \quad (3)$$

where $v_{jt} = R_{jt} - R_{bt}$

$$u_{mt} = R_{mt}^* - R_{bt}$$

R_{bt} = a weighted average of market's borrowing and lending rate as defined by Brennen (1971).

R_{mt}^* = the true market rates of return, it is not observable.

Both v_{jt} and u_{jt} are unobserved, we can only observe y_{jt} and x_{jt} . The relationship between the true values and the observed values can be defined as

$$y_{jt} = v_{jt} + \delta_{jt}, \quad (4)$$

$$x_{mt} = u_{mt} + \eta_{mt},$$

where $\delta_{jt} = R_{bt} - R_{Tt} = E + \tau_{jt}$, $\eta_{mt} = E + \tau_{jt} - (R_{mt}^* - R_{mt})$,

$$R_{mt}^* - R_{mt} = \psi - \lambda_t, \quad \tau_{jt} \sim N(E, \sigma_\tau^2), \quad \text{cov}(\tau_{jt}, \eta_{mt}) = \sigma_\tau^2,$$

$$\eta_{mt} \sim N(E - \psi, \sigma_\eta^2), \quad \sigma_\eta^2 = \sigma_\tau^2 + \sigma_\lambda^2,$$

E and ψ are constant measurement errors of R_{bt} and R_{mt}^* respectively. .

R_{Tt} = monthly treasury bill rate,

R_{mt} = the market rate of return calculated from NYSE average

According to Lee and Jen (1978), E is always greater than zero and the sign of ψ is ambiguous. δ_{jt} and λ_t are random measurement errors of R_{bt} and R_{mt}^* respectively. The errors-in-variables model of this kind is

not entirely consistent with the classical case discussed by Johnston (1972) and others. See Chang and Lee (1977) and Fogler and Ganapathy (1982) for detail.

IV. Justification of the Mixed Measurement Error and the Random Coefficient Model

The best reason for using a random instead of fixed coefficient model has been explored by Hildreth and Houck (1968), where they argued that the random coefficient assumption captures a coefficient variation of the coefficient associated with omitted variables. Some multi-index models, i.e., the three-factor and four-factor models discussed in the appendix A and the multi-factor model developed by Sharpe (1977) can be used to justify the necessity of a random coefficient model for estimating the betas. Hildreth and Houck (1968) have employed two examples to justify the necessity of random coefficient models, i.e., (a) response of a plant to nitrogen fertilizer, and (b) response of household to the level of income. For case (a) the random coefficient assumption was used to capture the variation of regression coefficients associated with the omitted factors, e.g., temperature and rainfall. For case (b) the random coefficient assumption can be used to take care of the variation of the coefficients associated with the omitted demographic factors.

The specification analysis developed by Theil (1971) can be used to demonstrate Hildreth and Houck's arguments of using random coefficient model. If a multi-index rates of return generating procedure for jth security (or portfolio) is defined as

$$y_{jt} = b_j x_t + c_1 z_{1t} + c_2 z_{2t} + \dots + c_n z_{nt} + \epsilon_{jt} \quad (5)$$

where z_{1t} , z_{2t} , ... and z_{nt} are some omitted variables.

If z_{1t} , z_{2t} , ... and z_{nt} are statistically insignificant, then equation (5) reduces to the observable form of equation (3). The relationship between the estimated \hat{b}_j from an equation without specification errors (\hat{b}_j) and the estimated \hat{b}_j from an equation with specification errors (\hat{b}_j^e) can be defined as

$$\hat{b}_j^e = \hat{b}_j + \hat{A}_1 \hat{C}_1 + \hat{A}_2 \hat{C}_2 + \dots + \hat{A}_n \hat{C}_n \quad (6)$$

where A_1 , A_2 , ... and A_n are auxiliary coefficients between x_t and the omitted variables z_{1t} , z_{2t} , ... and z_{nt} . If \hat{A}_1 , \hat{A}_2 , ... and \hat{A}_n are all statistically not different from zero. Then \hat{b}_j^e will be an unbiased estimator for \hat{b}_j . However, \hat{b}_j^e is no longer an efficient estimator for \hat{b}_j . If some of the estimated \hat{A}_i 's are statistically significantly different from zero, then \hat{b}_j^e may also be a biased estimator for \hat{b}_j unless the biases caused by misspecification are cancelled by each other. In sum, the random coefficient market model instead of the fixed coefficient market model is a more general model for estimating betas.

If the excess market return increases by one per cent, all other factors remaining constant, the excess return for the j th security may respond randomly with a certain mean and a positive variance. Thus (3) can be rewritten as (7).

$$v_{jt} = b_{jt} u_{mt} + \varepsilon_{jt} \quad (7)$$

$$E \begin{pmatrix} \varepsilon_{jt} \\ b_{jt} \end{pmatrix} = \begin{pmatrix} 0 \\ \beta_j \end{pmatrix}, \quad V \begin{pmatrix} \varepsilon_{jt} \\ b_{jt} \end{pmatrix} = \begin{pmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix} \quad t=1, \dots, n$$

$$E \left(\begin{pmatrix} \varepsilon_{jt} \\ b_{jt} - \beta_j \end{pmatrix} [\varepsilon_{ji}, b_{ji} - \beta_j] \right) = 0 \quad \text{for } t \neq i$$

Equation (7) can be rewritten in fixed coefficient form as

$$v_{jt} = \beta_j u_{mt} + \varepsilon_{jt}^* \quad (8)$$

where $\varepsilon_{jt}^* = \varepsilon_{jt} + (b_{jt} - \beta_j)u_{mt}$.

Following Theil and Mennes (1959) and Theil (1971), the most efficient estimator of b_{jt} is

$$\hat{b}_{jt} = \frac{\sum_{t=1}^n \frac{u_t v_t}{\sigma_0^2 + \sigma_1^2 u_t^2}}{\sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 u_t^2}} \quad (9)$$

Since u_t and v_t are unobserved, we should substitute for them by x_t and y_t respectively. Therefore, the observed b_{jt} can be estimated by

$$\hat{b}'_{jt} = \frac{\sum_{t=1}^n \frac{x_t y_t}{\sigma_0^2 + \sigma_1^2 x_t^2}}{\sum_{t=1}^n \frac{x_t^2}{\sigma_0^2 + \sigma_1^2 x_t^2}} \quad (10)$$

Implications of equation (10) can be found in the appendix B.

When both σ_0^2 and σ_1^2 are not known, following Theil and Mennes (1959), they can be estimated by using the residuals obtained from ordinary least squares and x_t . Under these circumstances, both $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ are affected by the measurement errors of y_t and x_t . The effects of measurement errors in both x_t and y_t on $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ and $\hat{\beta}_j$ will be analyzed in the following section.

V. Impact Measurement Errors on the Estimated Random Coefficient Parameters

Fabozzi and Francis (1978), Sunder (1980), and Lee and Chen (1980) have used the random coefficient model developed by Theil and Mennes (1959) to investigate the random nature of the beta coefficient in terms of the market model. However, they have entirely neglected the possible impacts of measurement errors associated with R_m and R_F on the estimated beta coefficients. Roll (1969, 1977) and Lee and Jen (1978) have argued that both R_m and R_F are measured with errors and therefore, x_{mt} as indicated in equation (5) is measured with errors.

Following Theil (1971), Francis and Fabozzi (1978), and Lee and Chen (1980), the model used to estimate the parameters σ_0^2 and σ_1^2 as indicated in equation (10) can be defined as

$$\hat{\epsilon}_{jt}^{*2} = \sigma_0^2 P_t + \sigma_1^2 Q_t + f_{jt} \quad (11)$$

where $\hat{\epsilon}_{jt}^*$ is the estimated ordinary least square (OLS) residual from equation (8); f_{jt} is the residual term for the multiple regression of equation (19). Finally, P_t and Q_t are defined as

$$(A) \quad P_t = 1 - u_t^2 / \sum u_t^2 \quad (12)$$

$$(B) \quad Q_t = u_t^2 \cdot [1 - 2(u_t^2 / \sum u_t^2)^2 + u_t^4 / (\sum u_t^2)^2]$$

If the sample size is large enough, then equation (12) can be approximately defined as [See Theil and Mennes (1959) for detail]

$$\hat{\epsilon}_{jt}^{*2} = \sigma_0^2 + \sigma_1^2 u_t^2 + f_{jt} \quad (13)$$

However, both v_{jt} and u_{mt} are not observable and y_t and x_t are used to replace them to estimate $\hat{\epsilon}_{jt}^{*2}$, σ_0^2 and σ_1^2 . Therefore, the RCEV CAPM can be defined as

$$y_{jt} = \beta_j x_t + [-\beta_j \eta_{mt} + (b_{jt} - \beta_j)x_t + \epsilon_{jt}] \quad (14)$$

Under this circumstance, it is easy to show that the multiple regression to be used to estimate the random coefficient parameters can be rewritten as

$$\hat{\epsilon}_{jt}^2 = (\sigma_0^2 + \beta_j^2 \sigma_\eta^2) + \sigma_1^2 x_t^2 + [-\sigma_1^2 \eta^2 - 2\sigma_1^2 \eta + f_{jt}] \quad (15)$$

where $\hat{\epsilon}_{jt}$ is the estimated OLS residuals of equation (14).

Equation (15) implies that the expected value of $\hat{\delta}_{jt}^2$ can be defined as

$$E(\hat{\epsilon}_{jt}^2) = \sigma_0^2 + \beta_j^2 \sigma_\eta^2 + \sigma_1^2 \{E(x_t^2) - \eta^2\} \quad (15')$$

where σ_0^2 is the pure OLS residual variance; $\beta_j^2 \sigma_\eta^2$ is the variance associated with the measurement errors of excess market rates of return; $\sigma_1^2 \{E(x_t^2) - \eta^2\}$ is the variance associated with the random coefficient systematic risk. It is clear that σ_0^2 is the nonsystematic risk. However, $\beta_j^2 \sigma_\eta^2$ and $\sigma_1^2 \{E(x_t^2) - \eta^2\}$ are not necessarily nonsystematic risks. Therefore, the standard OLS regression two-component risk decomposition technique can no longer apply to the RCEV CAPM.

From appendix C the impacts of measurement errors associated with R_m and R_F on the estimated σ_1^2 is now analyzed. If $\lambda = 2$, then $(\hat{\sigma}_1^2 / \sigma_1^2) \sim 64$. This result implies that Fabozzi and Francis's (1978), Saunder's (1980), and Lee and Chen's (1980) estimates of $\hat{\sigma}_1^2$ are potentially biased downward. Cochran (1970) has shown that the measurement errors of

regressors can reduce the coefficient of determination (\bar{R}^2) for a regression. This argument has also implied that the t values and \bar{R}^2 's associated with Francis and Fabozzi's (1978), Sunder's (1980) and Lee and Chen's (1980) empirical results are also downward biased.

Now, the bias associated with the systematic risk estimate for the random-coefficient-errors-in-variable CAPM as defined in equation (12) is analyzed.

Equation (10) can be approximately rewritten as⁵

$$\hat{b}_{jt} = \frac{\sum_{t=1}^n x_t y_t [1 - \frac{\sigma_1^2}{\sigma_0^2} x_t^2 + \dots] / n}{\sum_{t=1}^n x_t^2 [1 - \frac{\sigma_1^2}{\sigma_0^2} x_t^2 + \dots] / n} \quad (10')$$

If σ_1^2 approaches zero, then (10') reduces to fixed coefficient type of errors-in-variables model. If σ_1^2 is not negligible, then the second term for both numerator and denominator is investigated in detail as

$$(A) \quad \sum_{t=1}^n x_t y_t [- \frac{\sigma_1^2}{\sigma_0^2} x_t^2] / n = - \frac{\sigma_1^2}{\sigma_0^2} \sum_{t=1}^n x_t^3 y_t / n \quad (16)$$

$$(B) \quad \sum_{t=1}^n x_t^2 [- \frac{\sigma_1^2}{\sigma_0^2} x_t^2] / n = - \frac{\sigma_1^2}{\sigma_0^2} \sum_{t=1}^n x_t^4 / n$$

If the individual firm's rates of return are independent of the third moment of market rates of return, then [16A] is negligible. However, the term associated with (16B) is still an important term in estimating systematic risk.

Substituting equations (G) of appendix C and the measured errors variance of market rates return into (16B). We have

$$\text{plim} \left[- \frac{\hat{\sigma}_1^2}{\hat{\sigma}_0^2} \sum_{t=1}^n x_t^4 / n \right] = -3 \frac{\sigma_1^2}{\hat{\sigma}_0^2} [\sigma_u^4 (1-\lambda^2)^2] \quad (17)$$

where σ_u^2 is the variance of true market rates of return as defined in equation (7), and $\lambda = \sigma_n^2 / \sigma_u^2$.

If the quality of market index is relatively high, i.e., the magnitude of λ is relatively small, then $\frac{\sigma_1^2}{\hat{\sigma}_0^2} [\sigma_u^4 (1-\lambda^2)^2]$ approaches $\frac{\sigma_1^2}{\sigma_0^2} (\sigma_u^4)$, and under this circumstance, the estimated systematic risk of the RCEV market model will be similar to those obtained from the standard random coefficient market model.

From the above-mentioned analysis, it can be concluded that the bias of estimated systematic risk obtained from the random coefficient CAPM may well be higher than the bias of estimated systematic risk obtained from the fixed coefficient CAPM unless the measured errors of market rates of return and risk-free rate are trivial. The main purpose of using random coefficient regression model to estimate the slope is to improve the efficiency. If the regression is measured without errors, then both fixed coefficient and random coefficient estimator of slope are unbiased and consistent. However, the random coefficient estimator is more efficient than the fixed coefficient estimator. If the market rates of return are measured with errors, then it can also easily be proved that the random coefficient systematic risk estimator

is more efficient than the fixed coefficient systematic estimate.⁶

However, the random coefficient estimator is subject to bias.

From equations (10') and (16), we have

$$\begin{aligned}
 \hat{b}'_{jt} &= \left[\sum_{t=1}^n X_t Y_t / n \right] / \left[\sum_{t=1}^n X_t^2 / n - \frac{\hat{\sigma}_1^2}{\hat{\sigma}_0^2} \left(\sum_{t=1}^n X_t^4 / n \right) \right] \\
 &= \left[\sum_{t=1}^n Y_t X_t / \sum_{t=1}^n X_t^2 \right] \left[\frac{1}{1-k} \right] \\
 &= \left[\hat{\beta}_j^T - \frac{h}{1+h} \hat{\beta}_j^T \right] \left[1 + \frac{k}{1-k} \right] \\
 &= \hat{\beta}_j^T - \frac{h}{1+h} \hat{\beta}_j^T + \hat{\beta}_j^T \left(\frac{k}{1-k} \right) - \left(\frac{h}{1+h} \right) \left(\frac{k}{1-k} \right) \hat{\beta}_j^T
 \end{aligned} \tag{18}$$

where $\hat{\beta}^T$ = estimated time beta coefficient for jth firm

$$k = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_0^2} \left(\frac{\sum_{t=1}^n X_t^4}{\sum_{t=1}^n X_t^2} \right)$$

$$h = \frac{\hat{\sigma}_n^2}{\hat{\sigma}_u^2}$$

Equation (18) implies that the beta coefficient obtained from the RCEV CAPM can be decomposed into four components, i.e., (i) true component, (ii) bias due to measurement errors, (iii) bias due to specification bias and (iv) interaction bias. This is the most generalized beta estimator. The relationship between this kind of beta estimate and other beta estimates is analyzed as follows:

(a) If both market rates of return and risk-free rate are free from measurement errors, i.e., $\lambda = 0$, then equation (33) reduces to

$$b'_{jt} = \hat{\beta}_j^T + \hat{\beta}_j^T \left(\frac{k}{1-k} \right) \quad (19)$$

This is the random coefficient beta estimate obtained by Fabozzi and Francis (1980), Sunder (1980) and Lee and Chen (1980).

(b) If either the CAPM or the market model is not subject to specification bias, then equation (18) reduces to

$$b'_{jt} = \hat{\beta}_j^T - \frac{h}{1+h} \hat{\beta}_j^T \quad (20)$$

This is the beta estimate obtained by Roll (1969), Lee and Jen (1978) and others

(c) If either the CAPM or the market model is free from both specification bias and measurement errors, then equation (18) reduces to

$$b'_{jt} = \hat{\beta}_j^T \quad (21)$$

This is the "ideal" beta estimate. This analysis implies that the true ideal beta instead of the estimated random coefficient beta (b'_{jt}) should be used to estimate the cost of capital and investment analysis. Therefore, the method of determining the $\hat{\beta}^T$ for individual security (or portfolio) will be an important subject for further research.

VI. Conclusions

The mean square error [MSE] measure can generally be used as a criteria to determine either the fixed coefficient beta estimate or the random coefficient beta estimator is more appropriate. Empirically, the historical estimates of systematic risk are generally used to forecast the future systematic risk estimates in the investment analysis and port-

folio management. Klemkosky and Martin (1975) have used the mean squared errors [MSE] criteria in determining the power of alternative systematic risk forecasting models. The MSE can be decomposed into (i) the square of bias, (ii) efficiency and (iii) sampling errors. Lee and Chen (1982) have used Klemkosky and Martin's MSE decomposition technique to test whether the fixed-coefficient systematic risk on the random-coefficient systematic risk should be empirically used to forecast future betas, and have found that the random coefficient beta estimates perform better than the fixed coefficient beta estimates in beta forecasting. Hence, Lee and Chen's empirical results lend some support to using the MSE criteria to determine whether random coefficient or fixed coefficient beta estimates are more appropriate.

The results related to the impact of measurement errors on the quadratic regression coefficient estimate as indicated in equation (G) of Appendix C can also be used to re-examine on other well-known empirical studies of finance research. First, Fama and Macbeth (1973) have used the four-factor model as indicated in equation (2) of appendix (A) to test the efficiency of the capital market. If the estimated systematic risk for an individual security is normally distributed, then the square of the estimated systematic risk will follow a χ^2 distribution. Therefore, $\bar{\beta}_j^2$ also follows a χ^2 distribution. The estimated β_j and $\bar{\beta}_j^2$ are generally measured with errors, and therefore, the estimated γ_{2t} is a downward biased estimator. Secondly, Treynor and Mazuy (1965) have added a square term to the CAPM to test whether a mutual fund's rates of return generating process is linear or not. If the market rates of return used by Treynor and Mazuy are

measured with errors as discussed in this study, then the estimated coefficient associated with their empirical results are generally downward biased.

Footnotes

¹The fixed coefficient regression model is traditionally used to estimate the betas. Most recently Fabozzi and Francis (1978), Sunder (1980) and Lee and Chen (1980) have shown that the random coefficient regression model can be used to improve the efficiency of beta estimates.

²Often in the literature the term market model and CAPM are used interchangeably. Actually, they are not interchangeable. Fama (1976) and Scott and Brown (1980) have suggested the model defined in equation (2) should be called MMRPF.

³Brennan's (1971) model implies that the intercept is zero. If estimation based upon the null hypothesis that the Brennan model is true, the inclusion of the intercept will lead to inefficiency. Therefore, the intercept is assumed to be zero.

⁴Lee and Chen (1980) have used this type of specification to estimate random coefficient betas. Fabozzi and Francis (1978) do not use a risk-premium type of market model to estimate betas. Both Fabozzi and Francis (1978) and Lee and Chen (1981) have used Theil (1971) random coefficient procedure to estimate random coefficient betas. The market model used by Sunder (1980) is identical to that used by Fabozzi and Francis. However, Sunder has generalized Theil's random coefficient estimation procedure to meet his market model.

⁵Divide both numerator and denominator of equation (10) by $n\sigma_0^2$, we have

$$b_{jt} = \frac{\frac{n}{\sum_{t=1}^n x_t^2 y_t / n(1 + \frac{\sigma_1^2 x_t^2}{\sigma_0^2})}}{\frac{n}{\sum_{t=1}^n x_t^2 / n(1 + \frac{\sigma_1^2 x_t^2}{\sigma_0^2})}} \quad (a)$$

$$\text{since } \frac{\sigma_1^2 x_t^2}{\sigma_0^2} < 1, \text{ therefore, } \frac{1}{(1 + \frac{\sigma_1^2 x_t^2}{\sigma_0^2})} \quad \text{can} \\ \frac{2}{\sigma_0^2}$$

be approximately written as

$$\frac{1}{1 + \frac{\sigma_1^2 x_t^2}{\sigma_0^2}} = (1 - \frac{\sigma_1^2 x_t^2}{\sigma_0^2} + \dots) \quad (b)$$

substituting equation (b) into (a), we have equation (10') as indicated in the text. If x_t is measured without errors, then $\text{plim } b_{jt}^* = \beta_j$ as indicated in equation (A) of appendix B.

⁶Following Theil (1971, 623).

It can easily be shown that the variance of the estimator from (10') is smaller than that of equation (FA) in appendix (B).

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Appendix A Alternative Multifactor Models

(A) The two-factor model, which is due to Black (1972) can be defined as

$$R_{jt} = R_{zt} (1 - \beta_j) + \beta_j R_{mt} + \epsilon_{jt}, \quad (1)$$

where R_{zt} is called zero beta factor and is defined as the return on a portfolio which has a zero covariance with R_{mt} .

(B) The four-factor model, which is due to Fama and Macbeth (1973) can be written as

$$R_{jt} = \gamma_{0t} + \gamma_{1t} \beta_j + \gamma_{2t} \bar{\beta}_j^2 + \gamma_{3t} \bar{\sigma}_j(U) \quad (2)$$

where γ_{0t} plays the same role as R_{zt} ,

$\bar{\beta}_j^2$ is the average of the β_j^2 for all individual securities in portfolio j.

$\bar{\sigma}_j(U)$ is the average of the residual standard deviations from the market model for all securities in portfolio j.

(C) The three-factor model, which was developed by Merton (1973) can be defined

$$E(R_j) = R_F + \lambda_1 [E(R_m) - R_F] + \lambda_2 [E(R_N) - R_F], \quad (3)$$

where R_N = the return on the asset which is perfectly negatively correlated with changes in the riskless interest rate.

ρ_{Nm} = the correlation coefficient between R_N and R_M .

$$\beta_{jm} = \frac{\text{cov}(R_j, R_m)}{\sigma_m^2}$$

$$\beta_{jN} = \frac{\text{cov}(R_j, R_N)}{\sigma_N^2} \quad \beta_{Nm} = \frac{\text{cov}(R_N, R_m)}{\sigma_m^2}, \quad \beta_{MN} = \frac{\text{cov}(R_N, R_m)}{\sigma_N^2}$$

$$\lambda_1 = \frac{(\beta_{jm} - \beta_{jN}\beta_{Nm})}{1 - \rho_{Nm}^2} \quad \lambda_2 = \frac{(\beta_{jN} - \beta_{jM}\beta_{mN})}{1 - \rho_{Nm}^2}.$$

Equation (3) is a theoretical model but it can easily be shown that the rates of return generating process can be defined as

$$R_{jt} = \alpha_0 + \alpha_1 R_{mt} + \alpha_2 R_{Nt} + \epsilon_{jt}$$

Stone (1974) has derived a model similar to this equation.

Appendix B Analysis of Equation 10

When both σ_0^2 and σ_1^2 are known the relationship between $\text{plim } \hat{b}_{jt}$ and $\text{plim } \hat{b}'_{jt}$ can be derived as follows:

(i) Substituting (8) into (9) and taking the probability limit of b_{jt} , we have

$$\text{plim } \hat{b}_{jt} = \frac{\text{plim } \beta_j \sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 u_t^2} + \sum_{t=1}^n \frac{\epsilon_{jt}^* u_t}{\sigma_0^2 + \sigma_1^2 u_t^2}}{\text{plim } \sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 u_t^2}} \quad (\text{A})$$

If $\text{plim } \sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 u_t^2}$ exists,

then $\text{plim } b_{jt} = \beta_j$.

Equation (A) implies that the OLS estimate of systematic risk is a consistent estimate if u is free from the measurement errors. Note that the expression of equation (9) should be divided by sample size n before taking plims. Similarly, these concepts should also be used in equations with plims in this appendix.

(ii) Substituting (4) into (10) and taking the probability limit of \hat{b}'_{jt} , then we have

$$\text{plim } \hat{b}'_{jt} = \frac{\beta_j + \frac{\text{plim} \sum_{t=1}^n \frac{\tau_t'^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}{\text{plim} \sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}}{1 + \frac{\text{plim} \sum_{t=1}^n \frac{\eta_t'^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}{\text{plim} \sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}} \quad (B)$$

where $\tau_t' = \tau_t - E$, $\eta_t' = \eta_t - E + \psi = \tau_t' + \lambda_t$.

Equations (A) and (B) can be used to estimate the bias of estimated systematic risk associated with the random-coefficient-errors-in-variables model as

$$\text{plim } \hat{b}'_{jt} - \text{plim } \hat{b}_{jt} = \frac{D(1-\beta_j) - \beta_j C}{1 + C + D} \quad (C)$$

where

$$C = \frac{\text{plim} \sum_{t=1}^n \frac{\lambda_t^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}{\text{plim} \sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}$$

$$D = \frac{\text{plim} \sum_{t=1}^n \frac{\tau_t^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}{\text{plim} \sum_{t=1}^n \frac{u_t^2}{\sigma_0^2 + \sigma_1^2 x_t^2}}.$$

Equation (C) implies that the OLS estimator is no longer a consistent estimator unless

$$D(1 - \beta_j) - \beta_j C = 0$$

This equation implies that

$$1 - \beta_j / \beta_j = -\frac{C}{D} = \frac{\text{plim} \sum \lambda_t^2 / \sigma_0^2 + \sigma_1^2 x_t^2}{\text{plim} \sum \tau_t^2 / \sigma_0^2 + \sigma_1^2 x_t^2}$$

If $\sigma_1^2 = 0$, then (15) reduces to the fixed coefficient errors-in-variables case, i.e.

$$\text{plim} \hat{b}_{jt} - \text{plim} \hat{b}_{jt} = \frac{(1 - \beta_j) \frac{\sigma_{\tau't}^2}{\sigma_{u_t}^2} - \beta_j \frac{\sigma_{\lambda t}^2}{\sigma_{u_t}^2}}{1 + \frac{\sigma_{\tau't}^2 + \sigma_{\lambda t}^2}{\sigma_{u_t}^2}}$$

$$= \frac{(1 - \beta_j) \sigma_{\tau't}^2 - \beta_j \sigma_{\lambda t}^2}{\sigma_{u_t}^2 + \sigma_{\tau't}^2 + \sigma_{\lambda t}^2} . \quad (D)$$

The relative magnitude between equation (C) and equation (D) can be used to determine whether the random coefficient or the fixed coefficient model should be used to estimate the systematic risk of CAPM. The derivation of this criteria will be done in the next section.

If $\sigma_0^2 = 0$, then (C) reduces to

$$\text{plim} \hat{b}_{jt} - \text{plim} \hat{b}_{jt} = \frac{D'(1 - \beta_j) - \beta_j C'}{1 + C' + D'} \quad (E)$$

$$\text{where } C' = \frac{\text{plim} \sum_{t=1}^n \frac{\lambda_t^2}{x_t^2}}{\text{plim} \sum_{t=1}^n \frac{u_t^2}{x_t^2}}.$$

$$D' = \frac{\text{plim} \sum_{t=1}^n \frac{\tau_t^2}{x_t^2}}{\text{plim} \sum_{t=1}^n \frac{u_t^2}{x_t^2}}.$$

Under the above mentioned two circumstances, it should also be noted that equation (10) reduces to

$$(A) \quad \hat{b}_{jt} = \frac{\sum_{t=1}^n y_t x_t}{\sum_{t=1}^n x_t^2}$$

$$(B) \quad \hat{b}_{jt} = \frac{\frac{\sum_{t=1}^n y_t}{n}}{\frac{\sum_{t=1}^n x_t}{n}}. \quad (F)$$

Essentially, (FB) is a combined ratio estimator. In other words, for $\sigma_0^2 = 0$ or $\sigma_1^2 = 0$, the knowledge of σ_0^2 and σ_1^2 is not required at all. Equation (FB) implies that the average ratio estimate will equal the ordinary least squares (OLS) estimate if the OLS residual variance is essentially due to the randomness of slope coefficient.

Appendix C Impact of Measurement Errors on Estimated σ_1^2

Following Griliches and Ringstad (1970), the variables, u_t , x_t , and η are parameterized as follows. It is assumed that u_t , x_t , and η are all normally distributed with zero means and variances σ_u^2 , σ_x^2 , and σ_η^2 .

We know that $\bar{x}_t = 0$, hence $\bar{u}_t = 0$. We can parameterize our problem such that $\sigma_x^2 = 1$, $\sigma_\eta^2 = \lambda < 1$, and hence $\sigma_u^2 = 1 - \lambda < 1$.

Thus

$$u \sim N(0, 1 - \lambda),$$

$$\eta \sim N(0, \lambda)$$

$$x \sim N(0, 1)$$

and therefore

$$x^2 \sim \chi^2(1, 2)$$

$$\eta^2 \sim \chi^2(\lambda, 2\lambda^2)$$

From Theil (1957, 1971), it can be shown that

$$E(\hat{\sigma}_1^2 - \sigma_1^2) = -\sigma_1^2 b_{\eta x}^2 - 2\sigma_1^2 b_{(un)x}^2 = \text{plim}(\hat{\sigma}_1^2 - \sigma_1^2) \quad (A)$$

where $b_{\eta x}^2$ and $b_{(un)x}^2$ are auxiliary regression coefficients. Moreover, given the above assumptions and definitions, we also have

$$\begin{aligned} \text{cov}(un)x^2 &= E(un)(u^2 + 2un + \eta^2) - (Eun)(Eun^2) \\ &= 2Eun^2 = 2(Eu^2\eta^2) = 2\lambda(1 - \lambda) \end{aligned} \quad (B)$$

and

$$\begin{aligned}\text{cov}(\eta^2 x^2) &= E(\eta^2)(u^2 + 2un + n^2) - (E\eta^2)[E(u^2 + 2un + n^2)] \\ &= E(\eta^4) - (E\eta^2)^2 = \text{Var}(\eta^2) = 2\lambda^2\end{aligned}\quad (\text{C})$$

Therefore

$$b\eta^2 x^2 = \frac{\text{Var}(\eta^2)}{2} = \lambda^2 \quad (\text{D})$$

$$b(un)(x^2) = \frac{\text{Cov}(un)x^2}{\text{Var}x} = \lambda(1 - \lambda) \quad (\text{E})$$

Substituting equations (D) and (E) into equation (A), we obtain

$$\begin{aligned}\text{plim}(\hat{\sigma}_1^2 - \sigma_1^2) &= -2\sigma_1^2\lambda(1 - \lambda) - \sigma_1^2\lambda^2 \\ &= -\sigma_1^2\lambda(2 - \lambda)\end{aligned}\quad (\text{F})$$

Equation (F) implies that

$$\hat{\sigma}_1^2 - \sigma_1^2(1 - 2\lambda + \lambda^2) = \sigma_1^2(1 - \lambda)^2 \quad (\text{G})$$

Equation (G) implies that the bias associated with $\hat{\sigma}_1^2$ is $(1 - \lambda)^2$

where λ is the fraction of error variance in the total variance of the observed variable. Thus, the problem of errors-in-variables is significantly more serious for the non-linear term since the bias associated with a linear term is only $(1 - \lambda)$ [See Griliches and Ringstad (1970)].

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